PATENT APPLICATION

of

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for

An Apparatus and Method for Optical Determination of Intermediate Distances

FIELD OF THE INVENTION

The present invention relates to an optical apparatus and method for optical ranging. More specifically, it relates to employing beams propagating in certain relationships to each other to determine distances to features, especially when the distances fall in an intermediate range from a few centimeters to a few meters.

BACKGROUND

Determination of distances to stationary or moving objects is an important measurement challenge encountered in many fields of science and technology. In some cases, the distances to

the objects of interest are macroscopic and can be expressed in kilometers or larger units. This is true, for example, in determining distances or ranging remote structures or moving objects, such as vehicles. In other cases, the distances to the objects of interest are microscopic and can be expressed in millimeters or smaller units. Such conditions are encountered, for example, when determining distances between micro-structures on a silicon wafer. The prior art teaches a great variety of techniques to measure distances over various ranges in numerous fields and applications, including robotics and machine vision. An overview of a number of these techniques is found in *"Where am I" Systems and Technologies for Mobile Robot Positioning*, J. Borenstein, H.R. Everett, and L. Feng, A.K. Peters, Ltd., University of Michigan for the Oak Ridge National Lab (ORNL) D&D Program, Published by Wellesley, MA, copyright April 1996.

In the present case, we are interested in determining distances that fall between the macroscopic and microscopic, e.g., distances on the order of a few centimeters or meters. More specifically, of particular interest are techniques that use optical beams and can perform accurate distance measurements in this intermediate distance range.

One of the approaches taught by the prior art is based on optical ranging cameras or range-finding camera systems. Some examples of such cameras and systems are described in U.S. Pat. Nos. 6,057,909; 6,034,716; 5,200,793 and by S. Christie, et al., in *Measurement Science and Technology 6*,

September 1995, pp. 1301-1308. These systems are too cumbersome when a distance to one or just a few particular points or objects needs to be measured and no image of the scene is required.

Another approach taught by the prior art is based on scanning systems that use beams to determine distance. Most of these systems use the time-of-flight or propagation delay time to derive distance to the object. Several examples of corresponding apparatus and methods are found in U.S. Pat. Nos. 6,710,859; 6,064,471; 6,057,910; 5,959,734; 5,831,717; 5,724,123; 5,648,852 and 5,477,461. More sophisticated approaches using scanning are discussed in greater detail in a paper by Johnny Park, et al., "Dual-Beam Structured Light Scanning for 3-D Object Modeling", Third International Conference on 3-D Imaging and Modeling, Proceedings 5/28/2001 - 6/1/2001, pp. 65-72.

Unfortunately, most of the prior art approaches using scanning beams are not suitable for use in simple and low-cost systems for accurate determination of medium-range distances to stationary or moving objects. Specifically, many of these techniques, including time-of-flight or propagation time delay, are not suitable or not sufficiently accurate for measuring distances in the intermediate distance range.

OBJECTS AND ADVANTAGES

It is an object of the present invention to provide a simple, easy-to-use and low-cost system for determining distances to a stationary or moving object. More specifically, it is an object of the invention to provide a simple apparatus and method to determine a distance to a feature with the aid of a small number of optical beams propagating in a certain relationship to each other. The method and apparatus are particularly well-adapted for determination of intermediate distances extending from a few centimeters to a few meters.

SUMMARY

An apparatus and a method optically determine a distance r to a feature from an origin or a center. In one embodiment the apparatus has a beam generation unit for launching a reference beam on a reference path and a first beam on a first path. The center from which distance r is determined is selected such that it is along a line of the reference path and not along a line of the first path. In other words a line of the reference path intersects the center and a line of the first path does not. A rotation mechanism rotates the reference path and the first path about the center. As they rotate, the reference beam moves over the feature at a reference time t_r and the first beam moves over the feature at a first time t_1. The apparatus has a determination unit for determining distance r between the center and the feature from angular velocity ω of the reference beam and first beam while they move over the feature and from times t_r, t_1. In particular, a transverse velocity v of the reference beam

over the feature is obtained from times t_r and t_1. Distance r is determined from angular and transverse velocities ω, v with the aid of the well-known equation:

$$r = \frac{v}{\omega}.$$

Angular velocity ω of the beams can be obtained from times t_r, t_1 in some embodiments. Alternatively, an angular velocity unit measures angular velocity ω of the reference beam over the feature and communicates it to the determination unit.

The reference path and the first path can have folded path portions depending on the construction and application of the apparatus. Similarly, the rotation mechanism can be simple or complex and can take advantage of one or more elements such as mirrors, refractive elements, diffractive elements and holographic elements as well as any combination of such elements.

In a preferred embodiment the reference path and the first path are in a common plane Σ. In other words, reference beam and first beam are coplanar as they propagate to the feature. It should be noted that reference and first paths do not need to be coplanar in all embodiments, especially when the feature is large.

The determination unit has a detector for detecting radiation or light from the reference beam and the first beam which has interacted with the feature. When the feature is a waveguide or a hole, then the detector can observe radiation of the reference and first beams directly as they move over the feature. In other words, determination unit can either detect the reference and first beams as they move over the feature or it can detect scattered portions of reference and first beams at some distance away from the feature.

In the preferred embodiment the beam generation unit has a reference source for launching the reference beam and a first source for launching the first beam. The sources can be dedicated or they can be part of an active array of sources such as an array of Vertical Cavity Surface Emitting Lasers (VCSELs). It is further preferred that the reference source and the first source have distinct generation modes so as to endow the reference beam and first beam with mutually distinguishing properties. For example, the distinguishing properties can be polarizations, wavelengths, temporal beam formats, intensities, or any of various well-known types of optical modulation. When using wavelength as the distinguishing property, the determination unit can use a reference detector and a first detector as well as a wavelength filter or arrangement of wavelength filters to detect the reference beam and the first beam. Appropriate filters are used to separate polarizations when the distinguishing property is polarization. Alternatively, the detection unit can use the same detector without filters when

the distinguishing property is a temporal beam format or intensity.

In the preferred embodiment, the beam generation unit additionally launches a second beam on a second path that also rotates about the center. The second path is chosen such that the center is along a line of the second path, or such that a line of the second path intersects the center. As it rotates, the second beam moves over the feature at a second time t_2. The determination unit determines from times t_r and t_2 the angular velocity ω of the beams as they move over the feature. In this embodiment the beam generation unit comprises a second source for launching the second beam. It is preferred that the second source possess a distinct generation mode for endowing the second beam with a distinguishing property such as wavelength, polarization, temporal beam format, intensity or modulation, which distinguishes the second beam from the reference beam and/or first beam. It is further preferred that second path be in common plane Σ shared by reference and first paths. Note, however, that reference, first, and second paths do not need to be coplanar in all embodiments, especially when the feature is large. More specifically, when the feature is a micro-structure, all beams are preferably coplanar, i.e., they are in common plane Σ. When the feature is a macro-structure, such as an edge of an object, then the beams do not need to be coplanar.

In some alternative embodiments an apparatus to determine distance r to the feature from the center can operate without launching any beams, instead relying on external or ambient radiation. Such an apparatus has a radiation detection unit for selectively detecting radiation on the reference path and on the first path. As before, a line along the reference path intersects the center and a line along the first path does not. The rotation mechanism rotates the reference path and the first path about the center. Radiation from the feature is detected on the reference path at a reference time t_r and on the first path at a first time t_1. The determination unit determines distance r from the center to the feature from transverse velocity v and angular velocity ω of the reference path as it moves over the feature. Transverse velocity v is obtained from times t_r, t_1. Angular velocity ω can be determined from times t_r, t_1. In other embodiments, angular velocity ω is obtained from an angular velocity measurement unit. In still other embodiments angular velocity ω is obtained by detecting radiation on a second beam path.

In some embodiments distance r from a center to a feature is determined by launching a reference beam on a reference path and a first beam on a first path. The center is selected to be collinear with the reference path and non-collinear with the first path. The reference and first paths are rotated about the center. They move over the feature at reference and first times t_r and t_1. Distance r is determined from angular velocity ω of the reference beam over the feature.

Transverse velocity v is obtained from times t_r, t_1. Among other alternatives, angular velocity ω can be obtained from times t_r, t_1 or by measuring it with an angular velocity unit. In still other alternative embodiments, the location of center C can be selected to be non-collinear with the reference path and non-collinear with the first path.

In accordance with the method, non-collinear folded path portions can be added on to the reference or first paths, depending on conditions and application. In a preferred method the reference and first paths are arranged in common plane Σ and are endowed with mutually distinguishing properties. Further, a preferred method employs a second beam launched on a second path chosen such that the center is along a line along the second path to determine angular velocity ω of the reference beam. More precisely, the second path is rotated along with reference and first paths about the center such that the second beam moves over the feature. The second beam passes over the feature at a second time t_2. Angular velocity ω is determined from times t_r, t_2.

It should be noted that times t_r, t_1 and t_2 mark the events when the respective optical paths move transversely across the feature and thus interact with the feature. An optical signal emanating from this interaction may take the form of any measurable changes in an optical characteristic, i.e., amplitude, phase, polarization, wavelength, etc. For example, the feature may comprise a retro-reflecting or highly scattering surface, thereby increasing the detected

signal to mark this event. In another cases, the feature has absorption properties, such that the event is marked by a decrease in optical signal. In still other cases, these event times could include the capturing and processing of images of the feature and the spots of light moving across the feature.

Alternative methods of the invention can determine distance r to the feature from the center without launching any beams, instead relying on external or ambient radiation. Such methods rely on radiation from the feature detected on a reference and first paths. The paths are arranged such that the center is along a line of the reference path and not along a line of the first path. In still other embodiments, the reference path and the first path can be arranged such that the center is no along either the line of the reference path or the first path.

The reader will develop an appreciation for the invention from the detailed description and the appended drawing figures.

BRIEF DESCRIPTION OF THE DRAWINGS

Fig. 1 is a schematic diagram illustrating the principles of operation of an apparatus according to the invention.

Fig. 2 is a three-dimensional view of an apparatus according to the invention employing a mirror for rotating the optical paths.

Fig. 3 is a cross-sectional side view of the apparatus of Fig. 2 in plane Σ.

Fig. 4A is a diagram of the detection unit belonging to the apparatus of Fig. 2.

Fig. 4B is a diagram of an alternative detection unit that can be used by the apparatus of Fig. 2.

Fig. 5 is a graph of the mirror rotation angle $\gamma(t)$ during sinusoidal oscillation.

Fig. 6 is a cross-sectional side view of another apparatus according to the invention in plane Σ.

Fig. 7 is a graph of the mirror rotation angle $\gamma(t)$ during sinusoidal and saw-tooth type oscillation.

Fig. 8 is a cross-sectional side view of an alternative apparatus according to the invention in plane Σ.

Fig. 9 is a cross-sectional side view of still another alternative apparatus according to the invention in plane Σ.

Fig. 10 is a cross-sectional side view in plane Σ of a portion of another apparatus according to the invention.

Fig. 11 is a three-dimensional diagram illustrating an elongate object employing an apparatus according to the invention.

Fig. 12 is a three-dimensional diagram of yet another alternative apparatus according to the invention.

Fig. 13 is a three-dimensional view of a portion of an apparatus using non-parallel beam paths.

Fig. 14 is a three-dimensional view of a portion of the apparatus of Fig. 13 using a different arrangement of non-parallel beam paths.

Fig. 15 is a three dimensional view of an apparatus employing three non-coplanar beam paths.

DETAILED DESCRIPTION

The present invention will be best understood by initially reviewing the schematic diagram in Fig. 1 illustrating the basic principles of an apparatus **10** according to one embodiment of the invention. Apparatus **10** has a beam generation unit **12** with a reference source **14** for launching a reference beam **16** of optical radiation **18**. Unit **12** also has a first source **20** for launching a first beam **22** of optical radiation **24**. Reference beam **16** is launched on a reference path **26** and first beam **22** is launched on a first path **28**.

A distance r is defined from a center C to a feature **30**. Feature **30** can be a micro-structure or a macro-structure and it can be permanent, temporary, fixed or moving. In the present case feature **30** is a micro-structure, and more precisely a scattering point that scatters radiation **18** and **24**.

Center C is selected such that it is along a line A of reference path **26** and not along a line B of first path **28**. In other words, center C is on a line A that is collinear with reference path **26**. Note that although in the figure center C is located at a point where reference beam **16**

actually propagates it can also be located at a point beyond the line segment corresponding to reference path **26.** In addition, center C is not on a line B that is collinear with first path **28.** Note that lines A and B can have any relationship to each other, including being parallel. In the present embodiment, lines A and B are in a common plane Σ and they are not parallel. Thus, reference path **26** and first path **28** are in plane Σ. Also, in the present embodiment, reference source **14** is located at center C.

Reference source **14** and first source **20** have mutually distinct generation modes for endowing reference beam **16** and first beam **22** with mutually distinguishing properties. In the present case sources **14, 20** are lasers that emit radiation **18, 24** at different wavelengths and the distinguishing properties are wavelengths. A detector **32** positioned at feature **30** uses wavelengths of radiation **18** and radiation **24** to differentiate beam **16** from beam **22.**

Apparatus **10** has a rotation mechanism **34** that rotates reference path **26** and first path **28** about center C. The rotation is performed such that reference beam **16** and first beam **22** move over feature **30.** Beams **16** and **22** pass over feature **30** at a reference time t_r and at a first time t_1. In the present embodiment, rotation mechanism **34** rotates paths **26, 28** at a constant angular velocity ω_c.

Apparatus **10** has a determination unit **36** for determining distance r between center C and feature **30** from a transverse

velocity v and an angular velocity ω of reference beam **16** as it moves over feature **30**. Transverse velocity v of the reference beam is obtained from times t_r and t_1. Angular velocity ω of reference beam **16** can also be obtained from times t_r, t_1 or from an angular velocity unit **38**. When used, angular velocity unit **38** is in communication with determination unit **36**. Unit **38** can obtain angular velocity ω by communicating with rotation mechanism **34** or through independent observation.

During operation, detector **32** detects beams **16** and **22** at times t_r and t_1 when they move over feature **30**. In Fig. 1 detector **32** is detecting first beam **22** at time t_1 and sending a corresponding signal **40** to determination unit **36**. After rotation through angle σ_2 detector **32** detects reference beam **16** and sends a corresponding signal **42** to determination unit **36**.

In the embodiment shown, feature **30** is at a distance r_2 from center C. For comparison, feature **30** is also shown at a shorter distance r_1 from center C. When feature **30** is at distance r_2 the angle through which rotation mechanism **34** has to rotate paths **26, 28** between detection of beam **22** and then beam **16** by detector **32** is σ_2. When feature **30** is at distance r_1 the angle through which rotation mechanism **34** has to rotate paths **26, 28** between detection of beam **22** and beam **16** is σ_1. Note that angles σ_1 and σ_2 are different. In fact, for any specific distance r between center C and feature **30**, angle σ has a unique value.

Rotation mechanism **34** requires different amounts of time to rotate paths **26, 28** through different angles σ as it rotates at constant angular velocity ω_c. Thus, a time interval **44** between times t_1 and t_r is unique for each value of angle σ. The equation for time interval **44** can be expressed as:

$$|t_1 - t_r| = \frac{\sigma}{\omega_c} \qquad \text{(Eq. 1)}$$

Thus, when rotating at constant angular velocity ω_c, interval **44** between times t_1 and t_r when feature **30** is at distance r_1 is σ_1/ω_c. On the other hand, when feature **30** is at distance r_2 time interval **44** is σ_2/ω_c. In addition, transverse velocity v at which reference beam **16** moves over feature **30** changes with distance r as follows:

$$v = \frac{r\sigma}{|t_1 - t_r|} = r\omega_c . \qquad \text{(Eq. 2)}$$

In the present case, transverse velocity v is v_1 at distance r_1 and v_2 at distance r_2.

In general, knowledge of the geometric relationship between reference path **26** and first path **28** (i.e., the relative positions of lines A and B and the position of center C) provides an equation that describes transverse velocity v of reference beam **16** moving over feature **30** as a function of distance r and of angular velocity ω. Therefore, knowledge

of angular velocity ω, the relationship between paths **26, 28** and times t_r, t_1 is sufficient to determine distance r. Since in the present embodiment angular velocity ω is constant, i.e., $\omega=\omega_c$, the interval between successive times t_r measures the period and can thus be used to obtain angular velocity ω_c. Now, knowledge of times t_r and t_1 is sufficient to determine distance r. Alternatively, when angular velocity ω is not constant, then its value as beams **16, 22** move over feature **30** should be measured by unit **38**.

Determination unit **36** uses angular velocity ω and transverse velocity v of reference beam **16** moving over feature **30** to compute distance r. More precisely, equation 2 is based on the relationship:

$$r=\frac{v}{\omega}. \qquad \text{(Eq. 3)}$$

The principles of invention can be used in a wide variety of apparatus and methods. Fig. 2 illustrates a three-dimensional view of an apparatus **50** according to the invention employing a flat mirror **52** for rotating the optical paths. Apparatus **50** has a beam generation unit **54** equipped with a reference source **56**, a first source **58** and a second source **60** for generating reference, first and second radiation **62, 64**, and **66**. Sources **56, 58** and **60** have optics **68, 70, 72** for collimating radiation **62, 64, 66** into reference, first and second beams **74, 76** and **78**.

Sources **56, 58, 60** have mutually distinct generation modes to endow radiation **62, 64, 66** with mutually distinguishing properties. Specifically, sources **56, 58, 60** emit at mutually distinct wavelengths such that radiation **62** has a first wavelength λ_1, radiation **64** has a second wavelength λ_2 and radiation **66** has a third wavelength λ_3. Sources **56, 58, 60** can be Vertical Cavity Surface Emitting Lasers (VCSELs), light emitting diodes (LEDs) or other suitable emitters. Note that sources **56, 58, 60** can take advantage of other distinct generation modes to take advantage of other distinguishing properties such as polarization or intensity of radiation **62, 64, 66** or the temporal format of beams **74, 76, 78** such as pulse repetition rate or any well-known optical modulation. Alternatively, radiation **62, 64, 66** can be endowed with a distinguishing property by optics **68, 70, 72** and/or other optical elements including but not limited to optical filters, choppers, multiplexers and polarization rotators. A person skilled in the art will appreciate that still other alternatives exist for making radiation **68, 70, 72** distinguishable.

Beam generation unit **54** uses its sources **56, 58, 60** to launch reference beam **74** on a reference path **80**, first beam **76** on a first path **82**, and second beam **78** on a second path **84**. A center C chosen in the center of mirror **52** lies along a line of path **80** and not along a line of first path **82**. More precisely, path **82** is parallel to reference path **80** and offset from it by a distance d. Second path **84** is chosen such that center C is collinear with it. Specifically,

second path **84** passes through center C and preserves an angle ε to reference path **80**. This is accomplished by ensuring that sources **56, 58, 60** emit reference and first beams **74, 78** in parallel at offset d and second beam **78** at angle ε to reference beam **74**.

The reflective action of mirror **52** redirects or folds paths **80, 82, 84**. Thus, paths **80, 82, 84** have non-collinear folded portions between sources **56, 58, 60** and mirror **52**. It is important to note that the non-collinear folded portions are not used in the calculation of a distance r from center C to a feature **94**.

Because incident and reflected angles are equal and mirror **52** is flat, offset d between beams **74** and **76** and angle ε between beams **74** and **78** are preserved on reflection. Note that in this embodiment paths **80, 82, 84** are in a common plane Σ, or, equivalently, beams **74, 76, 78** are confined to propagate in plane Σ.

A rotation mechanism **86** uses mirror **52** for rotating reference path **80**, first path **82**, and second path **84** about center C. Mechanism **86** has a driver **88** for rotating mirror **52** about an axis of rotation **90** collinear with center C. Mirror **52** is suspended on hinges **92A, 92B**. Mirror **52** has a mirror axis M.A. perpendicular to mirror surface, passing through center C and axis of rotation **90**. A reference axis R.A. defines the neutral or unrotated position of mirror **52**. Driver **88** rotates mirror **52** by introducing a time-varying mirror

rotation angle γ between mirror axis M.A. and reference axis R.A. about axis of rotation **90**.

Distance r is defined from center C to feature **94**. In the present case feature **94** is a macro-structure. More precisely, feature **94** is an edge of object **92** that scatters radiation **62, 64** and **66**. The point on edge **94** from which beams **74, 76, 78** scatter is called scattering point P_o. Thus, distance r is defined between center C and scattering point P_o. Note that object **92** can be stationary or moving, and temporary or permanent. Conveniently, the position of point P_o on edge **94** is described in world coordinates (X_o, Y_o, Z_o).

Apparatus **50** has a detection unit **96** for detecting radiation **62, 64, 66** produced when beams **74, 76, 78** move over and scatter from point P_o. Detection unit **96** intercepts scattered radiation **62, 64, 66** arriving from point P_o along path g. Detection unit **96** has a wavelength filter **98** that is sensitive to wavelengths λ_1, λ_2, λ_3 of radiation **62, 64, 66** of beams **74, 76** and **78**. In particular, filter **98** spatially separates radiation **62, 64, 66** according to wavelength and sends radiation **62** to reference detector **100**, radiation **64** to first detector **102** and radiation **66** to second detector **104**.

Detectors **100, 102, 104** are connected to a determination unit **106**. Determination unit **106** obtains a first time t_1, a second time t_2 and a third time t_3 when reference, first and second beams **74, 76, 78** move over edge **94** and radiation **62, 64, 66** scatters from point P_o toward detection unit **96**. Note that

times t_1, t_2 and t_3 correspond to detection signals produced by detectors **102, 104, 106** after propagation time delay Δt due to time-of-flight along path g.

During operation beam generation unit **54** launches beams **74, 76, 78** on reference, first and second paths **80, 82, 84.** Mirror **52** reflects beams **74, 76, 78** while preserving offset d and angle ε. At mirror rotation angle γ beams **74, 76, 78** are reflected such that the optical angles between incident and reflected beams **74, 76** are 2γ and the optical angle between incident and reflected beam **78** is $2(\gamma+\varepsilon)$.

Rotation mechanism **86** rotates reference, first and second paths **80, 82, 84** about center C. Specifically, driver **88** of mechanism **86** rotates mirror **52** to yield a time varying mirror rotation angle $\gamma(t)$. Preferably, rotation angle $\gamma(t)$ is varied in a continuous manner such that mirror **52** rotates about axis of rotation **90** at an angular velocity $\omega(t)_{mir.}$ that also varies in a continuous manner. In this embodiment driver **88** varies rotation angle $\gamma(t)$ in a sinusoidal manner as follows:

$$\omega(t)_{mir.} = \dot{\gamma}(t) = A\sin(\omega_{mir.}t) \qquad \text{(Eq. 4)}$$

where A is the amplitude of oscillation and the dot represents the first time derivative. Amplitude A is sufficiently large to ensure that beams **74, 76, 78** all move over point P_o during each up and down oscillation or swing of mirror **52.** Note that angular velocity $\omega(t)_{beam}$ of beams **74,**

76, 78 moving over point P_o is twice angular velocity $\omega(t)_{mir.}$ of mirror **52**. That is because optical rotation angles correspond to twice mirror rotation angle γ due to the law of reflection.

The cross-sectional side view of Fig. 3 taken in plane Σ illustrates how beams **74, 76, 78** move over point P_o and scatter from it during an upswing in the following order: second beam **78**, first beam **76**, and reference beam **74**. Thus, during the upswing times t_r, t_1 and t_2 occur in the corresponding succession: t_2, t_1 and t_r. In Fig. 3 scattering of second beam **78** at time t_2 occurs at mirror rotation angle γ_2. At a larger mirror rotation angle γ_n during the up swing all beams **74, 76, 78** have moved beyond point P_o and have already scattered.

A diagram of detection unit **96** in Fig. 4A shows the succession of times t_2, t_1 and t_r at which detection signals are obtained from detectors **100, 102, 104** by determination unit **106**. Determination unit **106** determines distance r to point P_o from times t_2, t_1 and t_r. The determination is made by a timing and computation unit **108**, which belongs to unit **106**. Unit **108** employs the following equations in its determination:

$$\omega_{beam} = \frac{\varepsilon}{|t_2 - t_r|}, \qquad \text{(Eq. 5)}$$

$$v_{beam} = \frac{d}{|t_1 - t_r|}, \text{ and} \qquad \text{(Eq. 6)}$$

$$r = \frac{v_{beam}}{\omega_{beam}} = \frac{d|t_2 - t_r|}{\varepsilon|t_1 - t_r|}, \qquad \text{(Eq. 7)}$$

where v_{beam} is the linear velocity and ω_{beam} is the angular velocity of reference beam **74** as it moves over point P_o of edge **94**. It should be noted that the measurement of distance r can be performed any time beams **74, 76, 78** move over and scatter from point P_o on edge **94**. For the measurement to be accurate the instantaneous linear and angular velocities v_{beam}, ω_{beam} should be substantially constant while beams **74, 76, 78** are moving over point P_o. In an alternative approach, the time intervals between times t_1, t_r and t_2, t_r can be expressed without taking their absolute values. This can be done to obtain additional information from the order in which beams **74, 76, 78** scatter. For example, the sign (positive or negative) of the time intervals can indicate whether distance r was determined on an up- or downswing of mirror **52**.

In the present embodiment, since mirror rotation angle $\gamma(t)$ varies in a sinusoidal manner, the condition of substantially constant instantaneous linear and angular velocities v_{beam}, ω_{beam} is met in a linear region **110** of the sinusoidal oscillation of mirror **52**. Linear region **110** corresponds to a distance of about 1.72A, as better shown by graph **112** of mirror rotation angle $\gamma(t)$ in Fig. 5. In fact, Fig. 5 illustrates a desirable situation where times t_r, t_1, t_2 all fall within linear region **110**.

Since equation 7 yields distance r from times t_r, t_1, t_2 at which optical angles are $2\gamma_r$, $2\gamma_1$, $2\gamma_2$ (since optical angle is twice mirror rotation angle γ) it is not necessary to supervise the angular velocity of mirror **52**. Note that a total time interval **114** during which times t_2, t_1 and t_r are measured is short in comparison to the time required to complete an up- or downswing.

The measurement of distance r can be performed during an upswing or downswing of mirror **52**. During the upswing the order of times is t_2, t_1, t_r while during a downswing the order reverses to t_r, t_1, t_2. In situations when object **92** is moving the oscillation time of mirror **52** should be short to enable monitoring of movement of object **92** based on changing distance r to edge **94**.

An alternative detection unit **116** that can be used in place of unit **96** is illustrated in Fig. 4B, where corresponding parts are referred to with the same reference numerals. Unit **116** has wavelength filters **118, 120, 122** whose passbands are chosen to only pass radiation **62, 64, 66** respectively. In other words, filter **118** has a passband at λ_1, filter **120** has a passband at λ_2, and filter **122** has a passband at λ_3. Three detectors **100, 102, 104** dedicated to receiving radiation **62, 64, 66** of scattered beams **74, 76, 78** are placed behind filters **118, 120, 122**. Optional amplifiers **124, 126, 128** indicated in dashed lines can be provided for amplifying the signals sent to unit **106**. The remaining portion of detection unit **116**, namely determination unit **106** and timing and

computation unit **108** operate as described above. Note that unit **116** is in the process of receiving scattered radiation **62, 64, 66** at times t_r, t_1, t_2 or in the order corresponding to a down swing of mirror **52**.

Fig. 6 illustrates another apparatus **130** for determining a distance r from a center C of a mirror **132** to a feature **134** of an object **136**. Apparatus **130** has a beam generation unit **138** for generating a reference, first and second beams **140, 142, 144**. In this embodiment unit **138** is an active array of three sources **146, 148, 150** emitting radiation **152, 154, 156**. Sources **146, 148, 150** are active Fabry-Perot type resonators.

Beam generation unit **138** uses sources **146, 148, 150** and optics **158** for launching beams **140, 142, 144** on reference, first and second paths **160, 162** and **164**. Paths **160, 162** and **164** have non-collinear folded path portions commencing and sources **146, 148, 150** and terminating at the points of reflection from mirror **132** like the embodiment shown in Fig. 2. Reference path **160** and second path **164** lie along lines that intersect with center C. Indeed, these lines intersect at center C. Meanwhile, first path **162** is along a line intersecting with center C. Center C is chosen at the center of mirror **132** and on its axis of rotation perpendicular to the paper plane. Optics **158** ensure that paths **160, 162** preserve an offset d while paths **160, 164** preserve an angle ε. Thus, reference and first beams **140, 142** propagate in parallel at offset d and second beam **144** propagates at angle ε to reference beam **140**.

The reflective action of mirror **132** redirects or folds paths **160, 162, 164**. Thus, paths **160, 162, 164** have non-collinear folded portions between sources **146, 148, 150** and mirror **132.** Because incident and reflected angles are equal and mirror **132** is flat, offset d between beams **140** and **142** and angle ε between beams **140** and **144** are preserved on reflection. Note that in this embodiment paths **160, 162, 164** are in a common plane Σ, or, equivalently, beams **140, 142, 144** are confined to propagate in plane Σ.

A rotation mechanism (not shown) uses mirror **132** for rotating reference path **160,** first path **162,** and second path **164** about center C. The mechanism accomplishes this by rotating mirror **132** about the axis of rotation collinear with center C. The rotation is performed such that reference, first and second beams **140, 142, 144** scatter from feature **134** or scattering point P_o thereon. Beams **140, 142, 144** scatter at reference, first and second times t_r, t_1, t_2, respectively. Beams **140, 144** reflect from the center of mirror **132.** Beam **142** reflects from another point at a distance x from the center of mirror **132,** where:

$$x = \frac{d}{\cos\gamma(t)} \qquad \text{(Eq. 8)}$$

and $\gamma(t)$ is the mirror rotation angle which is varied in time by the rotation mechanism.

Apparatus **130** has a determination unit **166** for determining distance r from reference, first and second times t_r, t_1, t_2. Unit **166** examines back-scattered radiation **152'**, **154'** and **156'** returning from point P_o along the original paths **160, 162, 164**. Apparatus **130** has three beam splitters **168, 170, 172** for guiding back-scattered radiation **152'**, **154'** and **156'** to its reference, first and second detectors **174, 176, 178** to generate corresponding detection signals.

In this embodiment no distinguishing property between radiation **152, 154** and **156** is required. That is because only back-scattered radiation **152', 154', 156'** returning along the paths **160, 162, 164** is deflected by corresponding beam splitter **168, 170, 172** to dedicated detector **174, 176, 178** and produces a corresponding detection signal. A person skilled in the art will recognize that elements such as apertures can be used to further improve performance by eliminating any stray back-scattered radiation that is not returning along the original path. For example, apertures placed by the detectors and focusing optics placed along the paths in confocal arrangements can be used for these purposes.

The detection signals produced by detectors **174, 176, 178** are amplified by amplifiers **180, 182, 184** and delivered to determination unit **166**. Unit **166** has dedicated channels **186, 188, 190** for registering amplified signals from detectors **174, 176, 178**. Unit **166** obtains reference time t_r, first time t_1 and second time t_2 when beams **140, 142, 144**, respectively,

scatter from point P_o from the amplified signals registered in channels **186, 188, 190**. Note that times t_r, t_1 and t_2 correspond to detection signals produced by detectors **174, 176, 178** after time delay Δt due to time-of-flight along paths **160, 162, 164**. From times t_r, t_1 and t_2 determination unit **166** determines distance r from center C to point P_o. The determination of distance r is made by a timing and computation unit **192**, which belongs to unit **166**. Unit **192** employs equation 7 in its determination.

Apparatus **130** is compact and can be used to measure distance r from center C to moving or stationary feature **134**. In this embodiment feature **134** is a structure that scatters radiation **152, 154, 156** at point P_o. More precisely, feature **134** is a micro-structure **156** corresponding to a texture, and still more specifically to an imperfection in object **136**. Object **136** is a surface. Imperfection **134** causes beams **142, 144, 146** to scatter as they move over it and thus change the intensity of back scattered radiation **152', 154', 156'**. Note that the texture of micro-structure **134** that alters the scattering of beams **142, 144, 148** can include properties such as surface roughness, embedded scattering centers, retroreflecting microspheres or any other textural features. It should also be noted, that certain micro-structures can become resolvable when the spot size of the beams is focused to an appropriately small size.

During operation rotation angle $\gamma(t)$ of mirror **132** is varied such that all three beams **140, 142, 144** move over or are

scanned across point P_o. The measurement of distance r can be performed any time beams **140, 142, 144** move across and scatter from point P_o. As remarked above, the linear and angular velocities v_{beam}, ω_{beam} should be substantially constant while beams **140, 142, 144** are moving over point P_o to provide an accurate measurement.

The variation of mirror rotation angle $\gamma(t)$ can be sinusoidal, linear, or it can be governed by some other continuous function. Fig. 7 illustrates an exemplary sinusoidal variation **194** of rotation angle $\gamma(t)$ and an exemplary saw-tooth variation **196** of rotation angle $\gamma(t)$ for comparison. Note that saw-tooth variation **196** is continuous and linear over its entire swing range from -A to A, i.e., 2A, while sinusoidal variation **194** is linear over 1.72A only. Thus, saw-tooth variation **196** can be used for detecting scattering point P_o over a wider angular range.

Fig. 8 illustrates a cross-sectional side view in plane Σ of another apparatus **200** for measuring a distance r from a center C to a feature **202**. Apparatus **200** has a beam generation unit **204** equipped with sources **206, 208, 210** for launching reference, first and second beams **212, 214, 216**, respectively. Reference, first and second paths **218, 220, 222** are provided for beams **212, 214, 216** to feature **202**. Reference path **218** and second path **222** lie along lines that intersect with center C. First path **220** is not along a line that intersects with center C. Paths **218, 220** or beams **212, 214** are parallel and preserve an offset d during propagation.

Paths **218, 222** or beams **212, 216** preserve an angle ε to each other during propagation. All paths **218, 220, 222** or beams **212, 214, 216** are contained in common plane Σ.

Apparatus **200** has a rotation mechanism **224** for rotating paths **218, 220, 222** about center C. Mechanism **224** uses any suitable elements such as mirrors, refractive elements, diffractive elements or holographic elements to perform the rotation. In addition, sources **206, 210** are rotated and source **208** is shifted as indicated by the arrows to preserve offset d and angle ε. In this embodiment the rotation and shift of the sources is performed in concert with the action of mechanism **224**. The mechanics for performing the rotation and shift of the sources are a part of mechanism **224**.

During the rotation, reference beam **212** moves over feature **202** at reference time t_r and first beam **214** moves over feature **202** at a first time t_1. Second beam **216** moves over feature **202** at a second time t_2. A determination unit **226** is provided for determining distance r between center C and feature **202** from times t_r, t_1, t_2. Unit **226** has a detector **228** for detecting scattered beams **212′, 214′, 216′** returning from point P_o along path g.

Beams **212, 214, 216** have mutually distinguishing properties such as different wavelengths, polarizations, temporal beam formats, intensities or modulations to render them mutually distinguishable to determination unit **226**. For this reason, sources **206, 208, 210** have mutually distinct generation modes

such that beams **212, 214, 216** are endowed with mutually distinguishing properties. The operation of apparatus **200** is analogous to the operation of the embodiment of Fig. 2.

Fig. 9 illustrates still another apparatus **230** according to the invention in a cross-sectional side view taken in plane Σ. Apparatus **230** uses an active array **232** of sources **234** to generate reference, first and second beams **236, 238, 240.** A rotation mechanism in the form of optic **242** rotates optical paths **244, 246, 248** on which beams **236, 238, 240** are launched about a center C. In this embodiment, sources **234** are not moved, rather the appropriate sources **234** of array **232** are turned on and off by a control mechanism **250.** Optic **242** can employ any suitable elements such as mirrors, refractive elements, diffractive elements or holographic elements to perform the effective rotation about center C.

Apparatus **230** is used for measuring a distance r between center C and a feature **252.** During operation, mechanism **250** switches sources **234** is succession such that beams **236, 238, 240** scatter from feature **252** at reference, first and second times t_r, t_1, t_2. A determination unit **254** is provided for determining distance r from times t_r, t_1, t_2. Unit **254** has a detector **256** for detecting scattered beams **236', 238', 240'** returning from feature **252** along path g. Unit **254** determines distance r with the aid of equation 7.

The advantage of apparatus **230** is that it requires no moving parts. Beams **236, 238, 240** have mutually distinguishing

properties such as different wavelengths, polarizations, temporal beam formats, intensities or modulations to render them mutually distinguishable to determination unit **254**.

Fig. 10 shows a cross-sectional side view in plane Σ of a portion of an apparatus **260** that employs a reference beam **262** and a first beam **264** for measuring distance r from a center C of a mirror **278** to an absorbing feature **266** on a reflective surface **267**. A beam generation unit **268** has two sources **270, 272** for launching reference and second beams **262, 264** on reference and second paths **274, 276** to feature **266**. Paths **274, 276** are both contained in plane Σ and preserve an angle ε to each other. Thus, angle ε is preserved between beams **262** and **264** while they propagate along paths **274, 276**. In addition, reference path **274** and second path **276** are both along lines that intersect with center C. No first beam is used in this embodiment.

Apparatus **260** has a mechanism **280** for rotating reference and second optical paths **274, 276** about center C at a known rate. In the present embodiment mechanism **280** includes mirror **278** and a mirror drive **282**. Mirror **278** is mounted such that beams **262, 264** propagating in common plane Σ reflect from center C that lies on an axis of rotation of mirror **278**.

Mirror drive **282** changes a mirror rotation angle γ of mirror **278** at a known rate. In the present embodiment, mirror drive **282** changes rotation angle γ periodically at a certain angular frequency $\omega(t)_{mir.}$. It is important that rotation angle γ

change over a sufficiently large range such that reference beam **262** and second beam **264** move over entire feature **266.** Note that the optical angles by which beams **262, 264** are reflected from mirror **278** are twice γ, i.e., 2γ and beam angular frequency $\omega(t)_{beam}=2\omega(t)_{mir.}$.

Apparatus **260** has a determination unit **284** for determining distance r from center C of mirror **278** to feature **266** from two references times t_{r1}, t_{r2} or from two second times t_{21}, t_{22} and the known rate of change of mirror rotation angle γ. Determination unit **284** has a detector **286** for determining times t_{r1}, t_{r2}, t_{21}, t_{22} from scattered beams **262′, 264′** detected by detector **286.** Since surface **267** is reflective and feature **266** is absorbing, scattered beams **262′, 264′** are detected by detector **286** while beams **262, 264** are moving over surface **267** but not when moving over feature **266.** Thus, during an upswing time t_{r1} corresponds to beam **262** moving from surface **267** onto feature **266,** i.e., crossing over edge **266A.** Time t_{r2} corresponds to beam **262** moving from feature **266** back onto surface **267,** i.e., crossing over edge **266B.** Similarly, times t_{21}, t_{22} correspond to beam **264** crossing edges **266A** and **266B.**

A distinguishing property between beams **262** and **264** is used to differentiate between scattered beams **262′** and **264′.** Unit **284** is also connected to mirror drive **282** to obtain the rate of change of mirror rotation angle $\gamma(t)$, i.e., angular velocity $\omega(t)_{mir.}$ of mirror **278,** which it multiplies times two to obtain angular velocity ω_{beam} of beams **262, 264.**

During operation unit **284** applies the following equation to derive distance r using reference beam **262**:

$$r = \frac{v_{beam}}{\omega_{beam}} = \frac{s_\perp}{\omega_{beam}\delta t}, \qquad \text{(Eq. 9)}$$

where $s_\perp$ is the component of length of feature **266** normal to beam **262**, i.e., $s_\perp = s\cos\theta$, where θ is the angle of incidence of beam **262** to feature **266** at its center. Meanwhile, δt is the time interval during which no signal is detected by detector **286**. This time corresponds to the interval between reference times t_{r1} and t_{r2} when scattered beam **262'** is detected by detector **286**. Note that equation 9 can be used to determine distance r using times t_{11} and t_{12} of beam **264**. In other words, only one of beams **262, 264** is required to determine distance r when $s_\perp$ and $\omega(t)_{beam}$ are known. When $s_\perp$ is not known, unit **284** can calculate it using both beams **262, 264** since angle ε and beam angular velocity $\omega(t)_{beam}$ are known. More specifically, a perpendicular distance covered by beam **262** between times t_{r1} and t_{11} or between times t_{r2} and t_{12} is:

$$d_\perp = r\varepsilon. \qquad \text{(Eq. 10)}$$

and thus transverse velocity v_{beam} is:

$$v_{beam} = \frac{d_\perp}{|t_{r1} - t_{11}|} \text{ or,} \qquad \text{(Eq. 11A)}$$

$$v_{beam} = \frac{d_\perp}{|t_{r2} - t_{12}|}. \qquad \text{(Eq. 11B)}$$

Equation 11A computes transverse velocity v_{beam} by triggering off edge **266A** and equation 11B computes transverse velocity v_{beam} by triggering off edge **266B** (during an up swing of mirror **278**). When angular velocity $\omega(t)_{beam}$ is constant either edge can be used for triggering. When angular velocity $\omega(t)_{beam}$ is not constant then transverse velocity v_{beam} can be averaged from equations 11A and 11B.

Once transverse velocity v_{beam} is known, $s_\perp$ is calculated from:

$$s_\perp = v_{beam}|t_{r1} - t_{r2}| \text{ or,} \qquad \text{(Eq. 12A)}$$

$$s_\perp = v_{beam}|t_{11} - t_{12}|. \qquad \text{(Eq. 12B)}$$

Note that equation 12A uses beam **262** to calculate $s_\perp$ while equation 12B uses beam **264**.

As feature **266** moves and as its component normal to beam **262** changes i.e., as r and $s_\perp$ change, δt changes as well. Beams **262, 264** can be used to measure both as r and $s_\perp$. Note that the ability to measure $s_\perp$ can be used for measuring distances between features as well. The addition of another beam propagating along a beam path non-collinear with center C allows apparatus **260** to operate without having to interrogate mirror drive **282** for angular velocity $\omega(t)_{mir.}$ of mirror **278** based on the principles explained above.

Fig. 11 illustrates an elongate object **300** employing an apparatus **302** according to the invention to determine a distance r to a feature **304**. Apparatus **302** uses a beam generation unit **305** mounted at a known height to launch a reference, first and second beams **306, 308, 310** on reference, first and second paths **312, 314, 316**. Paths **312, 316** are along lines that intersect with a center C (not shown) while path **314** is not. Also, paths **312, 314** preserve an offset d and paths **312, 316** preserve an angle ε. Moreover, all paths **312, 314, 316** are in a common plane Σ.

A rotation mechanism (not shown) uses a MEMs scanning mirror to rotate paths **312, 314, 316** about center C in the manner explained above. At the instant shown, the scanning mirror is sending reference beam **306** along reference path **312** at a scan angle σ_s to its center axis C.A.

A determination unit **318** is mounted near a top end **320** of elongate object **300**. Determination unit **318** has a detector (not shown) for detecting scattered beams **306', 308', 310'**. Determination unit **318** determines distance r from times t_r, t_1, t_2 when beams **306, 308, 310** scatter from feature **304** or portions thereof. At the instant shown, beam **310** is scattering from an edge of feature **304** and sending a scattered beam **310'** along path g to determination unit **318**. Scattered beam **310'** arrives at an angle τ to center axis C.A.

Elongate object **300** has a tip **322** at an origin O of object coordinates (X',Y',Z'). Object **300** is inclined with respect to axis Z' by an inclination angle θ_o. Feature **304** is described in coordinates (X_r,Y_r,Z_r). Knowledge of the location of center C in coordinates (X',Y',Z') and angle σ at times when feature **304** of known geometry is detected permits one to determine the position of feature **304** in coordinates (X',Y',Z'). In other words, coordinates (X_r,Y_r,Z_r) can be indexed by an offset vector D_{ro} to object coordinates (X',Y',Z'). For precise indexing it is preferable to determine distance r based on a statistically significant number of scattering points on feature **304**.

In some cases feature **304** can be detected while elongate object **300** executes a motion **324** corresponding to a movement **326** of top end **320** and a movement **328** of tip **322**. Under these conditions a pose of object **300** can be tracked with respect to feature **304**. Note that to accomplish this, plane Σ may need to be rotated about center axis C.A. to ensure that beams **306, 308, 310** move over feature **304** as object **300** is executing motion **324**.

Fig. 12 is a three-dimensional diagram of yet another apparatus **330** according to the invention. Apparatus **330** has reference, first and second sources **332, 334, 336** for launching reference, first and second beams **338, 340, 342** on reference, first and second paths **344, 346, 348**. Reference path **344** is along a line that intersects with a center C, as is second path **348**. First path **346** is not along a line that

intersects with center C. Paths **344** and **348** preserve an angle ε between each other. Paths **344** and **346** are nearly parallel, as explained below. Center C defines the apex of a conical surface. An intersection of the conical surface with a plane surface **350** is indicated in a dashed ellipse **352**.

A rotation mechanism (not shown) rotates paths **344, 346, 348** about center C while preserving their geometric relationship. More precisely, paths **344, 346, 348** are rotated about a center axis C.A. of the conical surface passing through the apex or center C. As a result, paths **344, 348** are confined to move on the conical surface and thus trace out a scan path on surface **350** coincident with ellipse **352**. Path **346** is offset from path **344** and almost parallel with it. As clarified by a section **354** of the conical surface path **346** cannot be both confined to the conical surface and parallel to path **344**. Therefore, an offset distance d between paths **344** and **346** is only nearly constant. However, since section **354** is a hyperbola that rapidly approaches its linear asymptote, choosing **346** to lie on the asymptote ensures that paths **344** and **346** are nearly parallel at surface **350** and distance d is nearly constant. Because of these geometrical reasons, a scan point P_1 produced on surface **350** by first beam **340** will generally be offset from elliptical scan path **352** followed by scan points P_r, P_2 produced on surface **350** by beams **338** and **342**.

During operation the mechanism rotates paths **344, 346, 348** at an angular velocity $\omega(t)_{beam}$ about center axis C.A. Thus,

beams **338, 340, 342** move over a feature **356** on surface **350** and scatter from it at times t_r, t_1, and t_2. A determination unit **358** uses a detector **360** to detect scattered radiation produced by beams **338, 340, 342** moving over feature **356.** From times t_r, t_1, and t_2 unit **358** determines a distance r from center C to feature **356.** Note that beams **338, 340, 342** are endowed with mutually distinguishing properties so that detector **360** can differentiate them. Also note that this embodiment can employ just beams **338** and **340** when angular velocity $\omega(t)_{beam}$ is known (e.g., from a separate measurement unit or from the rotation mechanism).

Apparatus **330** is useful when a number of features lie on elliptical scan path **352** and their distances r_i to center C need to be known. Since beams **338, 340** are only nearly parallel, the features should be sufficiently large that both scan points P_r, P_1 move over them at all inclinations of surface **350** that are of practical interest.

Fig. 13 is a three-dimensional view of a portion of an apparatus **360** using reference, first and second beams **362, 364, 366** launched on three non-parallel paths **368, 370, 372** from sources **374, 376, 378** belonging to a beam generation unit **380.** Reference path **368** is along a line A that intersects with a center C chosen in the center of a mirror **382** and on its axis of rotation **384.** First path **370** is along a line B that does not intersect with center C and intersects reference path **368** in its non-collinear folded path portion at an intersection point **386.** Reference path **368** and first

path **370** preserve an angle ε_1 between each other. Second path **372** intersects reference path **368** at center C and these two paths preserve an angle ε_2 between each other. All three paths **368, 370, 372** are in a common plane Σ (not shown) and have folded portions extending from sources **374, 376, 378** to the surface of mirror **382**.

Lines A and B collinear with paths **368, 370** are indicated in dashed lines. Lines A, B intersect at an intersection point **386'** that is a virtual image of point **386**. A distance r' between center C and point **386'** is indicated along line A.

Apparatus **360** uses distinct temporal beam formats as the mutually distinguishing properties between beams **362, 364, 366**. Specifically, a time of emission between pulses of radiation or format time τ_{format} is different for each beam. First beam **364** uses a long format time, reference beam **362** uses an intermediate format time and second beam **366** uses a short format time. In this embodiment, the pulses all have the same duration, but in alternative embodiments their duration can be varied.

To measure a distance r from center C to a feature, in the present case a scattering point P_o, mirror **382** is rotated by a rotation mechanism about axis **384** to thus rotate paths **368, 370, 372** about center C. This is performed such that beams **362, 364, 366** all move over point P_o. Mirror **382** rotates about axis **384** at an angular velocity $\omega_{mir.}$. Beams **362, 364,**

366 as well as point **386′** rotate about axis **384** at angular velocity $\omega_{beam}=2\omega_{mir.}$.

Beams **362, 364, 366** scatter from point P_o at times t_r, t_1, t_2 and a determination unit (not shown) determines distance r from these times. In this embodiment, the determination unit uses the following equation:

$$r = \frac{\varepsilon_1 r'}{\left(\frac{|t_r - t_1|}{|t_r - t_2|}\varepsilon_2 - \varepsilon_1\right)}. \qquad \text{(Eq. 13)}$$

Fig. 14 illustrates a different arrangement of reference, first and second paths **368, 370, 372** in apparatus **360**. In this arrangement, second path **372** intersects reference path **362** in the non-collinear folded path portion at a point **390.** Line D collinear with path **372** is indicated in dashes. Line D intersects line A at an intersection point **390′** that is a virtual image of point **390**. A distance r′ between center C and point **386′** is indicated along line A and a distance r″ between center C and point **390′** is also indicated along line A. In this embodiment, the determination unit uses the following equation to determine distance r:

$$r = \frac{\left(\frac{r'\varepsilon_1}{|t_r - t_1|} - \frac{r''\varepsilon_2}{|t_r - t_2|}\right)}{\left(\frac{\varepsilon_2}{|t_r - t_2|} - \frac{\varepsilon_1}{|t_r - t_1|}\right)}. \qquad \text{(Eq. 14)}$$

Note that angles ε_1 and ε_2 can be equal in the embodiments of Figs. 13 and 14. Also note that equation 13 can be derived from equation 14 for the special case when $r''=0$.

In yet another embodiment reference path **368** as shown in Fig. 14 can be eliminated to leave only first path **370** and second path **372**. In this case one of these two paths can be used as the reference path. Note that of these two remaining paths neither is along a line that intersects with center C. A person skilled in the art will recognize that equation 14 can be used to determine r in this embodiment.

Fig. 15 illustrates yet another alternative of apparatus **360** in which paths **368, 370, 372** are not contained in common plane Σ. In fact, each path **368, 370, 372** is in its own plane. The three planes are parallel. This alternative can be used when the feature to which distance r is measured is a macro-structure of sufficient width to permit all beams **362, 364, 366** to move over it.

It should be noted that the apparatus and method of invention can be practiced without providing active illumination or beams. For example, referring back to Fig. 6, ambient radiation on paths **160, 162, 164** propagating to detectors **174, 176, 178** rather than back-scattered radiation **152', 154', 156'** returning along those paths can be used by determination unit **166** for determining distance r. Note that in this case ambient radiation needs to be present and its

intensity level needs to be sufficiently high to permit reliable detection by detectors **174, 176, 178**.

The apparatus and method of invention are well-suited for determining intermediate distances ranging from a few centimeters to a few meters. The apparatus and method are convenient in applications where time-of-flight is not appropriate due to resolution or other limitations. However, the apparatus and method are not limited to measuring intermediate distances. In fact, the invention provides a simple, easy-to-use and low-cost system for determining distances to stationary or moving objects with as few as two optical beams.

It should be noted that when the locations and separations of features or scattering points are known then the method of invention can be used for calibration. In particular, the method can be employed to calibrate angular velocities $\omega(t)$ or rotation angles $\gamma(t)$ of mirrors or other elements such as refractive elements, diffractive elements and holographic elements employed in the rotation mechanisms. A person skilled in the art will recognize that the above embodiments are merely exemplary and that various other embodiments are possible. Therefore, the scope of the invention should be determined by the appended claims and their legal equivalents.